Optical and Quantum Electronics **31:** 1267–1276, 1999.

Characteristic equations for different ARROW structures

BIN LIU, ALI SHAKOURI* AND JOHN E. BOWERS
Department of Electrical and Computer Engineering, University of California, Santa Barbara, CA 93106, USA (E-mail: liu@opto.ucsb.edu)
**Permanent address: Jack Baskin School of Engineering, University of California, Santa Cruz, CA 95064, USA*

Received 22 October 1998; accepted 31 March 1999

Abstract. Based on radiation modes and phase relations in different ARROW structures, the characteristic equations are presented that can avoid root searching in the complex plane and find the effective index, loss and field profile easily. This simple model gives an accurate intuitive picture for low loss leaky modes and it can be used to design and optimize the low loss ARROW devices of practical interest.

Key words: ARROW, mode characteristics, optical waveguides, transfer matrix

1. Introduction

Antiresonant reflecting optical waveguides (ARROW) (Duguay *et al.* 1986) have attracted a great deal of interest during the past several years. Instead of total internal reflection in conventional waveguides, ARROWs use antiresonant reflection as the guiding mechanism. This gives ARROW some remarkable features that are used in many applications, such as remote couplers (Gehler *et al.* 1994), filters (Chu *et al.* 1996), and polarization splitters (Trutschel 1995). In order to design ARROW devices, the knowledge of the characteristics of ARROW modes is important. An approximate expression for the propagation constant and loss have been given by Baba *et al.* (1998) and Baba and Kokubun (1992). The equivalent transmission line (Jiang *et al.* 1989) and the transverse resonance method (Huang *et al.* 1992) have also been used to investigate the dispersion and loss. A rigorous numerical method is based on a well known transfer matrix method (Chilwell and Hodgkinson 1984; Kubica *et al.* 1990). Because of the leaky property of ARROW modes, the solutions must be found in the complex plane. The root searching in the complex plane could be time consuming and tedious, especially for optimization of multiple layer structures. In this paper, we will give a set of simple characteristic equations for ARROW structures. The root searching for the new equations needs to be carried only on the real axis and the error is negligible for low loss ARROW modes of practical interest (loss smaller than a few dB/cm). In section 2, characteristic equations of different ARROW structures are presented, which are based on radiation modes and the phase relation in ARROW devices. Section 3 examines the modal characteristics such as effective index, loss and mode profiles in most of the ARROW structures proposed up to now.

The results from our equations and the exact model are then compared. Furthermore, a new ARROW filter, which combines a conventional InP/InGaAsP waveguide and an AlGaAs/GaAs ARROW, is proposed and simulated in Section 4.

2. Characteristic equations

We consider a planar multilayer waveguide as shown in Fig. 1. The general solution of the wave equation in each layer is well known:

$$E_{y,j} = A_j \exp(k_j(x - x_j)) + B_j \exp(-k_j(x - x_j)) \tag{1}$$

where $k_j = \sqrt{\beta^2 - k_0^2 n_j^2}$, A_j and B_j are the complex field coefficients, k_0 is the free space wavenumber, $\beta = k_0 n_{\text{eff}}$ is the propagation constant, n_{eff} is the effective index and x_j is the position of layer j. By imposing the continuity of the field and its derivative for each interface, it is easy to find:

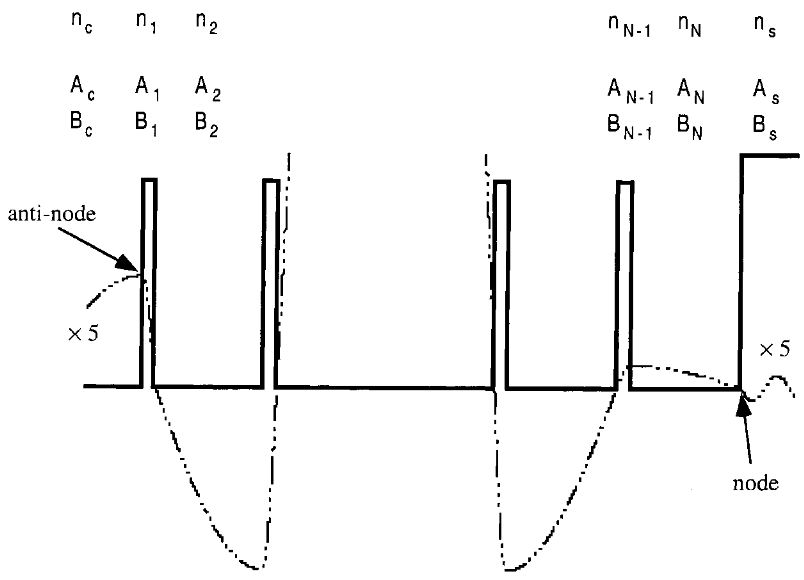

Fig. 1. Schematic diagrams of a planar ARROW structure and the field profile (dash line).

$$\begin{bmatrix} A_{j+1} \\ B_{j+1} \end{bmatrix} = T_j \begin{bmatrix} A_j \\ B_j \end{bmatrix}$$

where

$$T_j = \frac{1}{2} \begin{bmatrix} \left(1 + \zeta_j \frac{k_j}{k_{j+1}}\right) \exp(k_j d_j) & \left(1 - \zeta_j \frac{k_j}{k_{j+1}}\right) \exp(-k_j d_j) \\ \left(1 - \zeta_j \frac{k_j}{k_{j+1}}\right) \exp(k_j d_j) & \left(1 + \zeta_j \frac{k_j}{k_{j+1}}\right) \exp(-k_j d_j) \end{bmatrix}$$

d_j is the thickness of jth layer and

$$\zeta_j = \begin{cases} 1, & \text{TE} \\ n_{j+1}^2/n_j^2, & \text{TM} \end{cases}$$

So we can relate field coefficients in the cover (A_c and B_c) with the coefficients in the substrate (A_s and B_s):

$$\begin{bmatrix} A_s \\ B_s \end{bmatrix} = T \begin{bmatrix} A_c \\ B_c \end{bmatrix} \tag{2}$$

$$T = T_N \cdots T_1 T_c = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$$

For the guiding modes, the fields should be evanescent in the cap and the substrate layers, so $A_c = 0$ and $B_s = 0$ that results in the characteristic equation:

$$t_{11}(\beta) = 0 \tag{3}$$

For ARROW modes, since they are leaky, a characteristic equation $t_{11}(\beta) = 0$ can be found by assuming outgoing waves in the cover and substrate layers, with correct sign of k_c and k_s chosen (Chilwell and Hodgkinson 1984). The root β resides in the complex plane, so the root searching for structures with many layers and for optimization purposes could be time consuming and tedious. In the following, based on physical arguments, we will introduce a different characteristic equation for the radiation modes, that gives the ARROW mode effective indices on the real axis.

Radiation modes require both incoming and outgoing components to form standing waves in the substrate layer for one sided radiation modes, or in both cover and substrate layers for two sided radiation modes.

Equation (1) holds for radiation modes as well. Since the number of unknown variables is larger than the number of boundary conditions, a characteristic equation cannot be established. A simple relation between A_s (A_c) and B_s (B_c) will allow us to get a characteristic equation for the case of low loss ARROW modes. When the interference layers in ARROW waveguides satisfy the antiresonant condition, the reflectivity is very close to unity and the phase shift in each layer is 90°. This assures that the field at the outermost interface is a node or an anti-node. i.e., $A_s = \pm B_s$ or $A_c = \pm B_c$ for one sided radiation modes and $A_s = \pm B_s$, $A_c = \pm B_c$ for two sided radiation modes from Equation (1). The sign depends on the index of outermost two layers.

Now, let us look at each case separately.

(a) *One sided ARROW modes*: In one sided ARROW, the field in one side (cover) is evanescent $B_c = 0$ and in another side (substrate) is standing wave (actually the amplitude of the 'mode' is increased with the distance because of its lossy nature (Peierls 1979), i.e. $n_{\mathrm{eff}} > n_c$ and $n_{\mathrm{eff}} < n_s$. When the substrate index is larger than that of the last layer, $n_s > n_N$ (the right side of Fig. 1), the field in the outmost interface is a node, $A_s = -B_s$. From Equation (1) we get the characteristic equation:

$$t_{11} + t_{21} = 0 \tag{4}$$

When the index of substrate is smaller than that of the last film layer $n_s < n_N$, the field in the outmost interface is a anti-node (the left side of Fig. 1), $A_s = B_s$. So

$$t_{11} - t_{21} = 0 \tag{5}$$

(b) *Two sided ARROW modes*: In two sided ARROW, the field in both sides are standing waves, i.e. $n_{\mathrm{eff}} < n_c$ and $n_{\mathrm{eff}} < n_s$. There are three cases:

- $n_s > n_N$ and $n_c > n_1$. In this case, $A_s = -B_s$ and $A_c = -B_c$, so

$$t_{11} - t_{12} + t_{21} - t_{22} = 0 \tag{6}$$

- $n_s > n_N$ and $n_c < n_1$ (Fig. 1) or $n_s < n_N$ and $n_c > n_1$, $A_s = -B_s$ and $A_c = B_c$ or $A_s = B_s$ and $A_c = -B_c$, so

$$t_{11} + t_{12} + t_{21} + t_{22} = 0 \tag{7}$$

- $n_s < n_N$ and $n_c < n_1$, $A_s = B_s$ and $A_c = B_c$, so

$$t_{11} + t_{12} - t_{21} + t_{22} = 0 \tag{8}$$

Equations (3)–(8) are the characteristic equations for all kinds of ARROW structures. For all of these the effective index for guided (Equation (3)) and leaky modes (Equations (4)–(8)) for lossless materials can be found on the real axis. After finding the mode's effective index, it is simple to get its profile if we choose the correct A_c and B_c. Based on the first order perturbation theory, the loss of leaky modes can be found: $\alpha = 4.34k_0$ $\mathrm{Im}\left[t_{11}(n_{\mathrm{eff}})/t'_{11}(n_{\mathrm{eff}})\right]\mathrm{dB}/\mu\mathrm{m}$, where Im is the imaginary part, $t'_{11}(n_{\mathrm{eff}})$ is the differential with respect to n_{eff}.

3. Examples

In order to check the accuracy of our characteristic equations for different ARROW structures, we have compared the calculations with the exact model (Chilwell and Hodgkinson 1984; Kubica *et al.* 1990). Figs. 2(a)–(d) show the calculated effective index, loss and error as a function of the first interference layer thickness d_1 of an ARROW-A structure using our method and the exact model (Chilwell and Hodgkinson 1984; Kubica *et al.* 1990). Although this model is based on antiresonant condition, our computation shows that

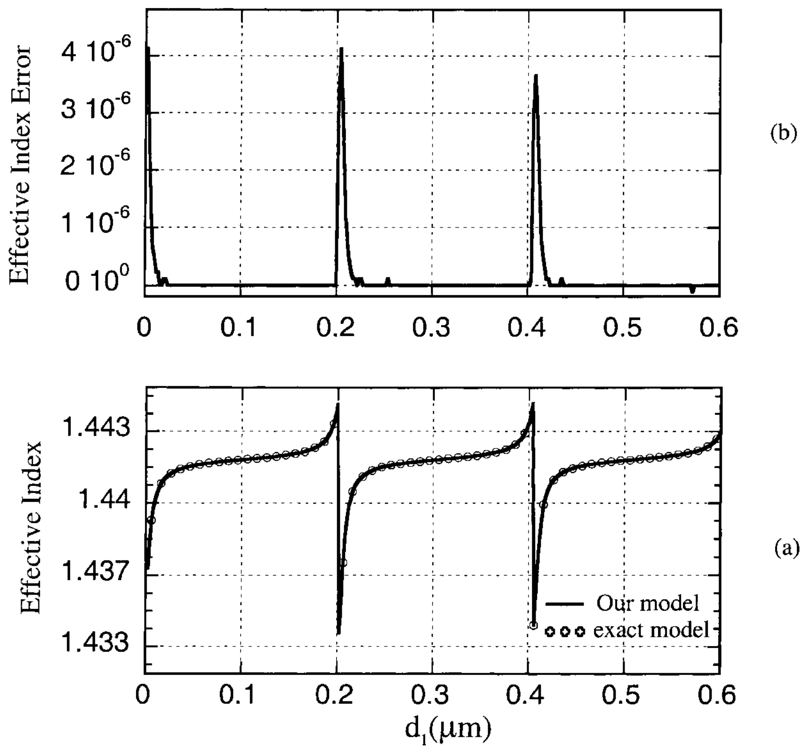

Fig. 2a,b

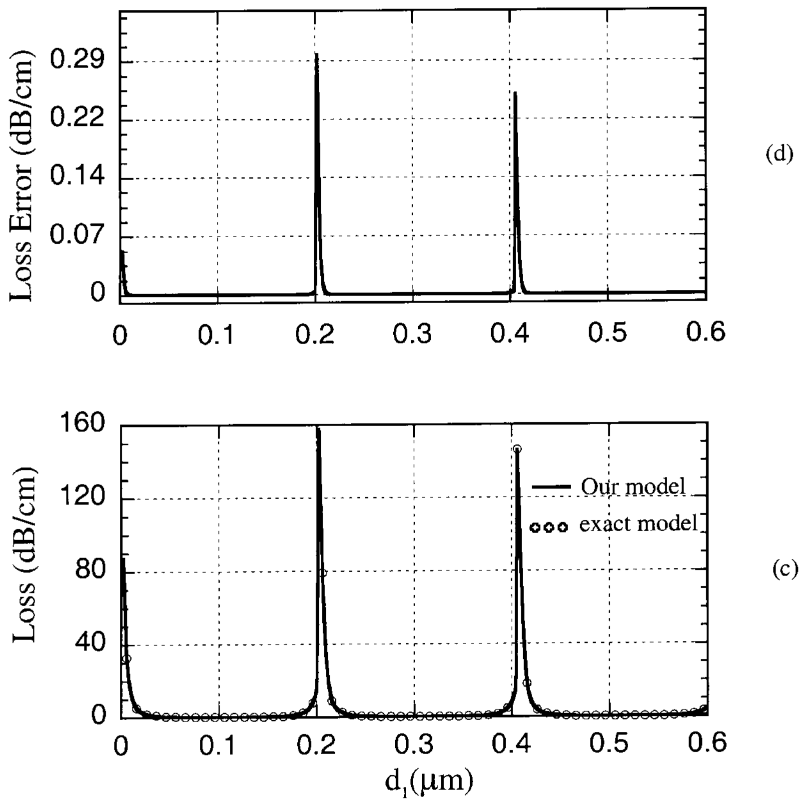

Fig. 2. The calculated effective index (a) and loss (c) of TE_0 mode of ARROW-A structure as a function of the thickness of the first antiresonant layer. (b) and (d) are the errors in our model comparing to the exact calculations.

antiresonance is not critical. Even when the thickness d_1 is far away from antiresonant condition, the current model matches the exact calculations very well (see Fig. 2). For example, when d_1 is around the resonant point where loss is the highest, the error between our method and the exact model is still smaller than 0.2% for TE1 mode. Table 1 displays the effective index and loss for several ARROW waveguides including ARROW-B (Baba and Kokubun 1992), two sided ARROWs and ARROW couplers (Chen and Huang 1996). We can see that for low loss ARROW modes, which are of main interest in practical applications, the approximate model with real roots in identical to the exact complex analysis.

4. InP/InGaAsP–AlGaAs/GaAs ARROW filter

In this section, we propose and design an ARROW filter (Fig. 3) which is based on two different waveguides: a conventional InGaAsP/InP waveguide

Table 1. Comparison of the effective index and loss between our model and exact model

Structure[a]	Characteristic equation	Mode	Effective index (this study)	Loss (dB/cm) (this study)	Effective index (exact)	Loss (dB/cm) (exact)
One sided	$t_{11} + t_{21} = 0$	TE1	1.4417085	0.25	1.4417085	0.25
ARROW-A		TE2	1.41798	270	1.4176	407
One sided	$t_{11} + t_{21} = 0$	TE1	1.5382528	0.11	1.5382527	0.11
ARROW-B		TE2	1.5336896	95	1.5336856	98
Two sided	$t_{11} - t_{12} + t_{21} - t_{22} = 0$	TE1	1.4578523	0.11	1.4578523	0.11
ARROW 1		TE2	1.4518589	76	1.4518454	98
Two sided	$t_{11} + t_{12} - t_{21} - t_{22} = 0$	TE1	3.1540497	0.53	3.1540497	0.53
ARROW 2		TE2	3.1393856	103	3.1393856	113
Two sided	$t_{11} + t_{12} + t_{21} + t_{22} = 0$	TE1	1.4578558	0.06	1.4578558	0.06
ARROW 3		TE2	1.4518726	49	1.4518551	58
ARROW	$t_{11} - t_{21} = 0$	Even	3.1541037	0.12	3.1541037	0.12
Coupler		Odd	3.1539980	0.14	3.1539980	0.14

[a] The structures (index, thickness (from the cap to substrate layers) and wavelength) of calculated ARROWs are listed below:

1. ARROW-A: $n = 1/1.45/3.5/1.45/3.5$; $d = \infty/4/0.1019/2.0985/\infty/\mu m$; $\lambda = 1.3\ \mu m$ (Jiang *et al.* 1989; Kubica *et al.* 1990; Huang *et al.* 1992).
2. ARROW-B: $n = 1/1.54/1.46/1.54/3.85$; $d = \infty/4/0.3/2/\infty/\mu m$; $\lambda = 0.633\ \mu m$ (Baba and Kokubun 1992).
3. Two sided ARROW 1: $n = 3.8/1.46/2.3/1.46/2.3/1.46/3.8$; $d = \infty/2/0.088/4/0.088/2/\infty/\mu m$; $\lambda = 0.633\ \mu m$.
4. Two sided ARROW 2: $n = 3.16/3.55/3.16/3.55/3.16/3.55/3.16/3.55/3.16$; $d = \infty/0.237/2/0.237/4/0.237/2/0.237/\infty/\mu m$; $\lambda = 1.55\ \mu m$.
5. Two sided ARROW 3: $n = 1.46/2.3/1.46/2.3/1.46/2.3/1.46/3.5$; $d = \infty/0.089/2/0.089/4/0.089/2/\infty/\mu m$; $\lambda = 0.6328\ \mu m$.
6. ARROW coupler: $n/1/3.16/3.55/3.16/3.55/3.16/3.55/3.16/3.55/3.16/$; $d/\infty/2/0.237/4/0.237/2/0.237/4/0.237/2/0.237/\infty/$; $\lambda = 1.55\ \mu m$ (Chen and Huang 1996).

and an AlGaAs/GaAs ARROW waveguide. Since ARROW has a very small waveguide dispersion (Duguay *et al.* 1986; Chu *et al.* 1996) and AlGaAs has a low material dispersion compared to a high material dispersion of InGaAsP around 1.55 μm, this structure can realize a narrowband filter. Another advantage of this structure is that ARROW's large mode allows efficient coupling with fibers. In order to make such a filter, the large lattice mismatch between InP and GaAs can be overcome with the use of wafer fusion technology (Liu *et al.* 1999).

To optimize the design of this ARROW, first we use the approximation equations in Baba *et al.* (1988) to get the initial d_1 (0.382 μm) and d_2 (2.204 μm), then use an iteration technique to find the optimal d_1 (0.406 μm) and d_2 (2.21 μm) for minimum loss. Since there is no root searching in the complex plane, the calculating time is reduced and the calculated value is the same as the rigorous model. For example, when $\lambda = 1.55\ \mu m$, $d_1 = 0.406\ \mu m$, $d_2 = 2.21\ \mu m$, starting with an initial effective index 3.24 and an initial searching step 0.00001, our model takes 18 s to

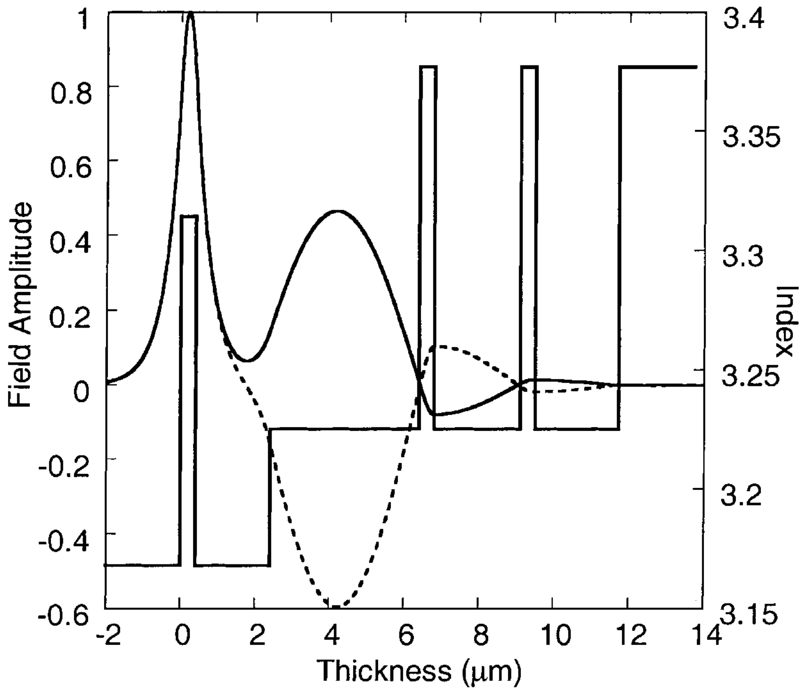

Fig. 3. The structure of InGaAsP/InP–AlGaAs/GaAs ARROW filter and the profiles of even and odd modes. The detailed structure is (from left to right): InP/$In_{0.81}Ga_{0.19}As_{0.4}P_{0.6}$ (0.403 μm)/InP (2 μm)/$Al_{0.4}Ga_{0.6}As$ (4 μm)/GaAs (d_1)/$Al_{0.4}Ga_{0.6}As$ (d_2)/GaAs (d_1)/$Al_{0.4}Ga_{0.6}As$ (d_2)/GaAs.

find the root with < le – 10 error, while the rigorous model takes 31 s on a PC (Pentium II, 266 MHz). The thickness of InGaAsP/InP waveguide is chosen to be 0.403 μm in order to satisfy the phase matching around 1.55 μm. The important characteristics of the filter are the dispersion curves of the two waveguides. Figs. 4 and 5 show the effective index and the loss of the separated waveguides and the even and odd supermodes of the coupled structure. As we expect from the design, the effective indices of two separated waveguides are the same at the phase matching point at 1.549 μm. Without coupling, the conventional InGaAsP/InP waveguide has no loss (neglecting the loss introduced by the material and process), and the AlGaAs/GaAs ARROW has a small loss, which increases slightly with the wavelength. When the two waveguides are coupled, the losses of even and odd modes of the coupler are the same at the phase matching condition. The field profiles of even and odd modes are also calculated and they are shown in Fig. 3. All these calculations match very well the rigorous model.

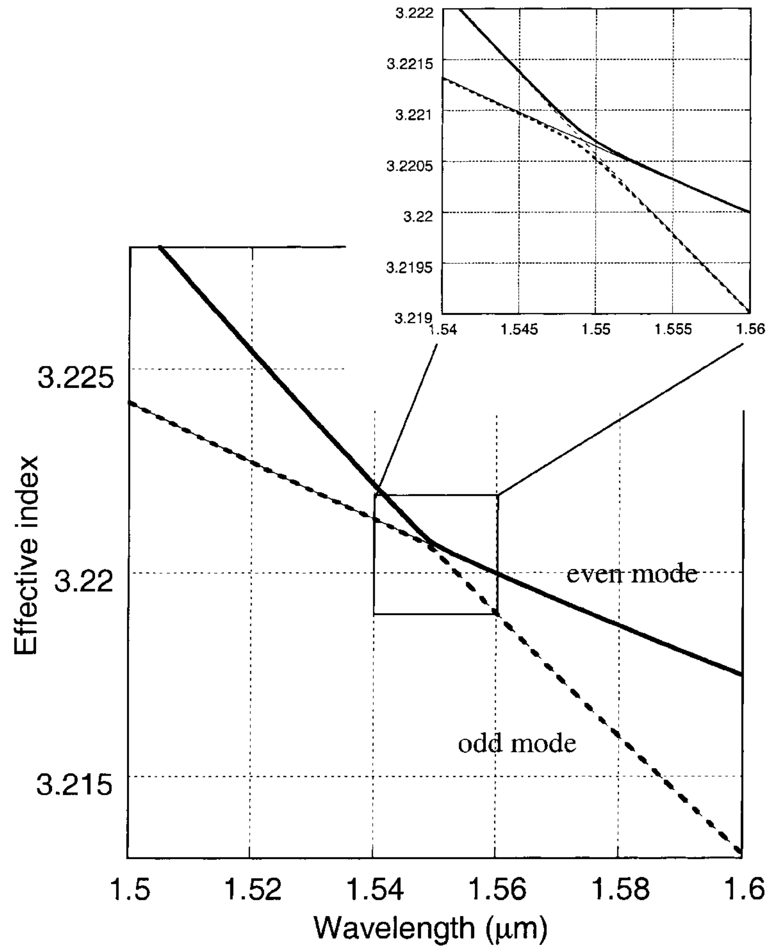

Fig. 4. The dispersion for the modes of the individual waveguides and the supermodes of the coupler filter. The calculations include the material dispersion.

5. Conclusion

In this paper, a set of simple and versatile characteristic equations is given for different ARROW structures. This method gives the precise effective index, loss and mode profile for low loss ARROW modes of practical interest (i.e. when their loss is smaller than a few dB/cm). This enables root searching in the real domain. The physical argument is simply based on the property of antiresonance and small loss, which means that the reflectivity at the outermost boundaries is near 1, and a phase relation existing for the field at the outermost layers. The use of these characteristic equations is demonstrated for different ARROW waveguides, couples and filter. This simple physical

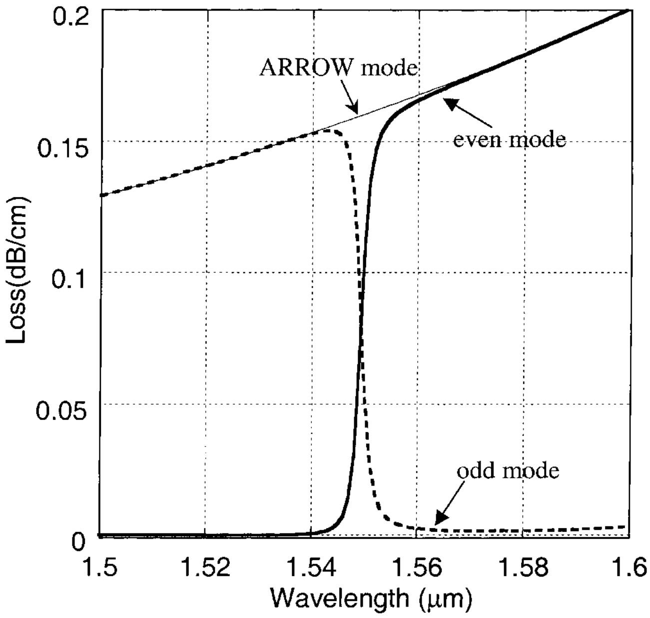

Fig. 5. The loss of even and odd supermodes of InGaAsP/InP–AlGaAs/GaAs ARROW filters. The top thin line is the loss of the separated AlGaAs/GaAs ARROW.

argument gives an accurate intuitive picture for loss leaky modes and it can be used to design and optimize ARROW devices.

References

Baba, T. and Y. Kokubun. *IEEE J. Quantum Electron.* **28** 1689, 1992.

Baba, T., Y. Kokubun, T. Sakakia and K. Iga. *J. Lightwave Technol.* **6** 1440, 1988.

Chen, Y.H. and Y.T. Huang. *J. Lightwave Technol.* **14** 1507, 1996.

Chilwell, J. and I. Hodgkinson. *J. Opt. Soc. Am. A* **1** 742, 1984.

Chu, S.T., M. Miura and Y. Kokubun. *IEEE Photon. Technol. Lett.* **8** 1492, 1996.

Duguay, M.A., Y. Kokubum and T.L. Koch. *Appl. Phys. Lett.* **49** 13, 1986.

Gehler, J., A. Brauer and W. Karthe. *Electron. Lett.* **30** 218, 1994.

Huang, W.P., R.M. Shubair, A. Nathan and Y.L. Chow. *J. Lightwave Technol.* **10** 1015, 1992.

Jiang, W., J. Chrostowski and M. Fontaine. *Opt. Commun.* **72** 180, 1989.

Kubica, J., D. Uttamchandani and B. Culshaw, *Opt. Commun.* **78** 133, 1990.

Liu, B., A. Shakouri, P. Abraham, Y. Chiu, S. Zhang and J.E. Bowers. *IEEE Photon. Technol. Lett.* **11** 93, 1999.

Peierls, R. *Surprises in theoretical physics*, Princeton University Press, Princeton, NJ, 1979.

Trutschel, U., F. Ouellete, V. Delisle, M.A. Duguay, G. Fogarty and F. Legerer. *J. Lightwave Technol.* **13** 239, 1995.